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Bayesian Inversion of Gamma Spectra

Chris Kunz

ABSTRACT

Today most gamma ray analysis techniques use the least squares method. This method does not take advantage of the information in the Compton Edge of the spectra. The gamma spectra data used in the project were collected at the Pacific Northwest National Lab (PNNL) in Hanford, Washington. Synthetic spectra, made using **SYNTH,** a program created at PNNL by Dr. W. Hensley, were employed. The development of a program capable of analyzing real data using a synthetic basis set was a major goal of the project. Several ways were identified to improve current gathering techniques, specifically concerning the characterization of individual detectors.

INTRODUCTION

Generally, society's ability to fully analyze data is well behind its ability to collect data. As a result, we spend millions of dollars, every year, designing hardware to collect radiation data that will not be processed optimally. This represents an enormous loss of money and information. Most current analysis techniques do not incorporate, in an information theoretic manner, noise or sensor error; as a result, much data are of marginal benefit (Boas, 1983). Our current techniques also fail when only small quantities of radioactive sources are present, a problem that is rapidly becoming more important in our society (Bauer, 1979). More emphasis must be placed on efficient information analysis if we wish to use the ever increasing amount of radioactive data being collected.

This project focused on processing information to obtain better results, for gamma ray spectra. The data processed consisted of a collection of complex gamma ray spectra, recent and archived, obtained with high resolution Ge detector arrays (Bauer, 1979).

The goal was to produce software that can be utilized throughout the national laboratory system to address nuclear nonproliferation, nuclear waste assessment, environmental cleanup, and medical applications. This software processes data, real-time, giving the user immediate feedback. The algorithm used in the program can account for noise and "smearing" effects caused by detection error. The intent is not to replace current processing methods and approaches, but to enhance them and to produce gamma spectra with finer resolution and higher signal-to-noise ratio.

MATERIALS AND METHODS

Because of the presence of noise in the model, the solution to the inverse problem is not unique and one has to take recourse to statistical methods to assign probability distributions for the source strengths $\{A_\alpha\}$. The math in this report was derived by Dr. K Noffsinger and Dr. R Inguva (Inguva & Noffsinger, 1994). Define $P\ (A_\alpha,\ldots,A_M|\vec{D},I)$ to be the probability distributions for $\{A_\alpha\}$ conditioned on the data D and any prior information I on $\{A_\alpha\}$. Once this quantity is obtained as a function of the source strengths, one can infer the most probable value for A_α by locating the maximum or the mean value defined by:

One can also estimate the goodness of $\langle A_\alpha \rangle$ by

$$\langle A_\alpha \rangle = \int (A_1,\ldots,A_M|\vec{D},I) A_\alpha \; dA_1 \ldots dA_M$$

computing the mean square deviation

$$\sigma^2_{A_\alpha} \equiv \langle A_\alpha^2 \rangle - \langle A_\alpha \rangle^2$$

where

$$\langle A^2 \rangle = \int P(A_1,\ldots,A_M|\vec{D},I) A_\alpha^2 \; dA_1 \ldots dA_M$$

Thus the solution to the inverse problem is given by the estimate, $\langle A_\alpha \rangle \pm \sigma_{A_\alpha}$, $\alpha = 1, \ldots, M$.

We now turn to the determination of the underlying probability density $P(A_1,\ldots,A_M|\vec{D},I)$ crucial to the development here. For this purpose we use Bayes' Theorem

$$P(A_1,\ldots,A_M|\vec{D},I) = P(A_1,\ldots,A_M|I) \left[\frac{P(\vec{D}|A_1,\ldots,A_M I)}{P(\vec{D}|I)} \right]$$

where $P(A_1,\ldots,A_M|I)$ is the "prior probability" in the absence of the data, $P(\vec{D}|A_1,\ldots,A_M,I)$ is the "sampling distribution" density for data $\vec{D}$ to contain evidence for the parameters $A_1,\ldots,A_M$ to be estimated, and $P(\vec{D}|I)$ is a normalization factor.

We now turn to a discussion of the various quantities on the RHS of Bayes' Theorem. The quantity $(\vec{D}|A_1,\ldots,A_M,I)$ is a prior probability density for the source amplitudes based on any prior information in the absence of the acquired data $\vec{D}$. Such information might consist of active emitters, etc. It also incorporates any other knowledge, for example, that all $A_1,\ldots,A_M$ must be positive semi-definite. The quantity $P(\vec{D}|I)$ is a normalization constant which will be dropped from further consideration. The quantity $P(\vec{D}|A_1,\ldots,A_M,I)$ is the "likelihood" or "sampling" distribution. Since the deviation of the data from the model is the additive noise, it is clear that:

$$P(\vec{D}|A_1,\ldots,A_M,I)=P(\vec{n}|I)$$

If the noise distribution is known, one can use it to infer $P(\vec{D}|A_1,\ldots,A_M,I)$. In practice, however, a Gaussian noise distribution works very well and will be used in this development. From a modeling and numerical analysis point of view a Gaussian distribution is easy to use. In a later section we will argue that, except under special circumstances, assumption of Gaussian distributions makes no significant difference in the estimation problem under consideration. Thus we write

$$P(\vec{D}|A_1,\ldots,A_M,I)=\left[2\pi\right]^{-\frac{N}{2}}\left[\prod_{i=1}^{N}\sigma_i\right]^{-1}e^{-\frac{\phi'}{2}}$$

$$\phi=\sum_{i=1}^{N}\frac{\left(d_i-\sum_\alpha A_\alpha g_{\alpha i}\right)^2}{\sigma_i^2}$$

where σ_i is the noise in the i^{th} channel and $g_{\alpha i}$ is the α^{th} source basis spectrum, i denotes channel value. With the above procedure we are now ready to define the estimation of $\{A_\alpha\}$. We begin by first rewriting the quantity Φ as

$$\phi=\sum_{i=1}^{N}\frac{d_i^2}{\sigma_i^2}-2\sum_{\alpha=1}^{M}h_\alpha A_\alpha+\sum_{\alpha,\beta}^{M}g_{\alpha\beta}A_\alpha A_\beta$$

where

$$(\tilde{h})_\alpha\equiv\sum_{i=1}^{N}\frac{(\tilde{g})_{\alpha i}d_i}{\sigma_i^2} \qquad g_{\alpha\beta}\equiv\sum_{i=1}^{N}\frac{(\tilde{g})_{\alpha i}(\tilde{g}^{\,T})_{i\beta}}{\sigma_i^2}$$

The matrix $\tilde{g}_{\alpha\beta}$, which is a function of the α^{th} and β^{th} source gamma spectra, is real symmetric. Since this matrix is in general not diagonal, we convert the RHS to a quadratic form by diagonalizing the $[g_{\alpha\beta}]$ matrix

$$\tilde{g}\tilde{u}_\alpha\equiv\gamma_\alpha\tilde{u}_\alpha \qquad \alpha=1,2,\ldots,M$$

The matrix of eigenvectors $\tilde{u}\equiv[\tilde{u}_1\tilde{u}_2\ldots\tilde{u}_M]$ provides the unitary transformation to diagonalize $\tilde{g}$, i.e.,

$$\tilde{u}^\dagger\tilde{g}\tilde{u}=\tilde{g}_d \qquad (\tilde{g}_d)_{\alpha\beta}=\delta_{\alpha\beta}\gamma_\alpha$$

The d subscript indicates a diagonal matrix. The third term is the definition of ϕ can therefore be written as

$$\tilde{A}^\dagger\tilde{g}\tilde{A}=\tilde{A}^\dagger\tilde{u}\tilde{g}_d\tilde{u}^\dagger\tilde{A}$$

Defining a transformed vector $\tilde{\xi}$ by

$$\xi_\alpha=\sum_\beta(\tilde{u}^\dagger)_{\alpha\beta}A_\beta$$

we have

$$\tilde{A}^\dagger\tilde{g}\tilde{A}=\tilde{\xi}^\dagger\tilde{g}_d\tilde{\xi}$$

Inverting the above relates $\tilde{A}$ to $\tilde{\xi}$ by $\tilde{A}=\tilde{u}\tilde{\xi}$. Using this result leads to

$$\phi=\sum_{i=1}^{N}\frac{d_i^2}{\sigma_i^2}-\sum_{\alpha=1}^{M}\bar{h}_\alpha\xi_\alpha-\sum_{\alpha=1}^{M}\bar{h}_\alpha^*\xi_\alpha^*+\sum_{\alpha=1}^{M}\gamma_\alpha\xi_\alpha^*\xi_\alpha$$

where $\bar{h}_\alpha\equiv\sum_\beta\sum_i(\tilde{u})_{\alpha\beta}(\tilde{g}^\dagger)_{\beta i}\frac{d_i}{\sigma_i^2}$ and $*$ denotes complex conjugate. Baye's Theorem allows us to switch the variables and the conditional statement since there is no prior information. Combining this with previous results provides

$$P(\xi_1,\ldots,\xi_M|\vec{D},I)=e^{-\frac{\phi_0}{2}}\left[2\pi\right]^{-\frac{N}{2}}\left[\prod_{i=1}^{N}\sigma_i\right]^{-1}e^{-\frac{\phi'}{2}}$$

where

$$\phi_0=\sum_{i=1}^{N}\frac{d_i^2}{\sigma_i^2}-\sum_{\alpha=1}^{M}\frac{\bar{h}_\alpha^*\bar{h}_\alpha}{\gamma_\alpha} \qquad \phi'=\sum_{\alpha=1}^{M}\left(\xi_\alpha^*-\frac{\bar{h}_\alpha^*}{\gamma_\alpha}\right)\left(\xi_\alpha-\frac{\bar{h}_\alpha}{\gamma_\alpha}\right)$$

We note that the probability density is a Gaussian in terms of the transformed amplitudes $\{\xi_\alpha\}$. It is clear that

$$<\xi_\alpha>=\frac{\bar{h}_\alpha}{\gamma_\alpha} \qquad <\xi_\alpha^*\xi_\beta>-<\xi_\alpha^*><\xi_\beta>=\delta_{\alpha\beta}\gamma_\alpha^{-1}$$

It then follows that

$$<A_\alpha>=\sum_\beta (\tilde{f})_{\alpha\beta}h_\beta \qquad (\tilde{f})_{\alpha\beta}=\sum_\gamma \frac{(\tilde{u})_{\alpha\gamma}(\tilde{u}^\dagger)_{\gamma\beta}}{\lambda_\gamma}$$

This result could have also been obtained directly by setting $\frac{\partial\phi}{\partial A_\alpha}=0$. In this sense the estimate for $<A_\alpha>$ is equivalent to the least squares solution. This conclusion is valid only when there is no prior information on $A_1,...,A_M$. The variance on the estimate $<A_\alpha>$ is given by

$$\sigma^2_{A_\alpha}\equiv<A^2_\alpha>-<A_\alpha>^2=(\tilde{f})_{\alpha\alpha}$$

$$<A_\alpha A_\beta>-<A_\alpha><A_\beta>=(\tilde{f})_{\alpha\beta}$$

Thus the inference on the amplitudes $A_1,...,A_M$ is summarized by

$$<A_\alpha>=\sum_\beta (\tilde{f})_{\alpha\beta}h_\beta\pm\sqrt{(\tilde{f})_{\alpha\alpha}}$$

The algorithm to obtain $\sigma^2_{A_\alpha}$ is straight forward. We start with the library of basis functions arranged vertically $\{g_\alpha;\ \alpha=1,...,M\}$ and then compute the S matrix (see below) given an estimate on the noise σ_i for the data value d_i. Assuming that the inverse exists, we calculate S^{-1}. Using the data and the noise vector σ_i^2, we can compute the vector $\tilde{h}$ and obtain estimates for $<A_\alpha>$ and the variance σ_α.

$$\sigma^2_{A_\alpha}=(S^{-1})_{\alpha\alpha} \quad ; \quad S_{\alpha\beta}\equiv\sum_i \frac{(\tilde{g})_{\alpha i}(\tilde{g}^{\,T})_{i\beta}}{\sigma_i^2}$$

RESULTS

Three basic types of data and source spectra were used in the experiment: synthetic data produced using SYNTH version 3.4 created by Pacific Northwest Laboratories, Richland WA; real data collected from Germanium detector number 3 in building 329 on the Hanford Site; and real data collected by the DOE in Fernald, Ohio using a NaI(Tl) cone penetrometer. The data was combined in various ways, synthetic source - synthetic data, real source - real data, synthetic source - real data, etc. Results are in the Appendix. Each Trial was assigned a unique number **(T#1,...,T#55)**. Definitions of table headers: IDEAL a's- ideal amplitudes of each source element; SRCE- the elements making up the source; Srce Amt- the amount of each source element in Decays/second unless otherwise noted; CNTS- the length of the source spectra; DATA AMT- amount of each source in the data; A's- the amplitude of each source predicted by the program; SIG A's- the corrected amplitude of each source; SIGMA1- uncertainty in the amplitude this value can be read as the first standard deviation; %ERROR- standard deviation divided by corrected A value times 100; STRPD a's- each source is stripped from the data one at a time based on the lowest SIGMA1 value, these are the amplitudes of each source at the time it is stripped.

DISCUSSION

Including information contained in the Compton Edge allows a more reliable prediction of source spectra. The Bayesian technique enables the algorithm to include prior information, further increasing the value of the program. Using synthetic data as source spectra to analyze real data spectra, is a very powerful option, and as the Synth program is updated the values will become better. To obtain the best results, however, means characterizing individual detector crystals. This means establishing a database of calibration spectra, under known conditions for each detector. After doing this, we can answer questions about weak sources in the presence of powerful sources and noise. It will also allow better analysis of trace amount sources, important for medical applications, nonproliferation, and environmental cleanup. This may be expensive to the user in time and money, but necessary to answer the new questions being asked.

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Appendix

IDEAL a's	SRCE	SRCE AMT	CNTS	DATA AMT	A's	SIG A's	SIGMA1	%ERROR	STRPD a's
T #1									
1.00	149Gd	1.0e+05	2048	1.0e+05	0.9996	0.9972	0.0768	7.7027	0.9972
1.00	153Sm	1.0e+05	2048	1.0e+05	0.9996	0.9963	0.1523	15.2881	0.9895
1.00	149Pm	1.0e+05	2048	1.0e+05	0.9940	0.9957	0.6102	61.2799	1.0104
1.00	132Cs	1.0e+05	2048	1.0e+05	0.9997	0.9987	0.0959	9.6041	1.0004
T #2									
1.00	149Gd	1.0e+05	2048	1.0e+05	0.9996	0.9972	0.0768	7.7027	0.9972
1.00	153Sm	1.0e+05	2048	1.0e+05	0.9996	0.9963	0.1523	15.2881	0.9895
1.00	149Pm	1.0e+05	2048	1.0e+05	0.9946	0.9957	0.6102	61.2881	1.0104
1.00	132Cs	1.0e+05	2048	1.0e+05	0.9997	0.9987	0.0964	9.6562	1.0004
0.00	133Ba	1.0e+05	2048		-0.0003	0.0000	0.0100	#DIV/0!	0.0000
T #3									
1.00	241Am	1.0e+06	4096	1.0e+06	1.0022	0.9984	0.0693	6.9393	0.9952
1.00	137Cs	1.0e+06	4096	1.0e+06	1.0002	1.0006	0.0346	3.4620	1.0008
1.00	60Co	1.0e+06	4096	1.0e+06	0.9997	0.9991	0.0245	2.4517	0.9991
0.00	132Cs	1.0e+06	4096		0.0000	0.0000	0.0000		0.0000
T #4	*the data spectrum is real*								
?	241Am	1.0e+06	4096	unknown	0.0025	0.0033	0.0040	122.6056	0.0039
?	137Cs	1.0e+06	4096	unknown	0.0257	0.0513	0.0054	10.5749	0.0562
?	60Co	1.0e+06	4096	unknown	0.0226	0.0343	0.0036	10.4021	0.0343
T #5	*the data spectrum is real, and contained an unknown isotope of Pu*								
?	232Pu	1.0e+05	4096	unknown	0	0	0.0000		.0e+00
?	233Pu	1.0e+05	4096	unknown	27	131	0.0000	0.0000	3.4e+02
?	234Pu	1.0e+05	4096	unknown	0	0	0.0000		.0e+00
?	235Pu	1.0e+05	4096	unknown	-29	-141	0.0000	0.0000	.0e+00
?	236Pu	1.0e+05	4096	unknown	2195	1889	0.0000	0.0000	.0e+00
?	237Pu	1.0e+05	4096	unknown	2	-3	0.0000	0.0000	.0e+00
?	238Pu	1.0e+05	4096	unknown	1676	2103	0.0000	0.0000	.0e+00
?	239Pu	1.0e+05	4096	unknown	1520	877	0.0000	0.0000	.0e+00
?	240Pu	1.0e+05	4096	unknown	340	124	0.0000	0.0000	.0e+00
?	241Pu	1.0e+05	4096	unknown	11074	10949	0.0000	0.0000	.0e+00
?	242Pu	1.0e+05	4096	unknown	17	-11	0.0000	0.0000	.0e+00
?	243Pu	1.0e+05	4096	unknown	1	1	0.0000	0.0000	.0e+00

IDEAL a's	SRCE	SRCE AMT	CNTS	DATA AMT	A's	SIG A's	SIGMA1	%ERROR	STRPD a's
?	244Pu	1.0e+05	4096	unknown	0	-8	big		.0e+00
?	245Pu	1.0e+05	4096	unknown	2	16	0.0000	0.0000	1.5e+01
?	246Pu	1.0e+05	4096	unknown	0	-1	0.0000	0.0000	.0e+00
T #6	*Cs131 was contributed to the data spectrum using the daughter feature in SYNTH, time=one half life of Ba131*								
1.00	131Ba	1.0e+05	2048	1.0e+05	0.4943	0.4939	0.0436	8.8255	0.4939
1.00	131Cs	1.0e+05	2048	1.0e+05	0.3575	0.3617	8.3568	2310.4279	-0.3586
T #7	*see comment for T#6*								
1.00	131Ba	1.0e+05	2048	1.0e+05	0.4943	0.4939	0.0436	8.8255	0.4939
1.00	131Cs	1.0e+05	2048	1.0e+05	0.3584	0.3618	8.3569	2309.8240	-0.3588
0.00	149Gd	1.0e+05	2048		0.0000	0.0001	0.0100	10000.0000	0.0001
T #8									
1.00	103Ru	1.0e+05	4096	1.0e+05	1.0018	0.9992	0.1375	13.7587	0.9957
1.00	99Mo	5.0e+04	4096	5.0e+04	1.0031	0.9980	0.1237	12.3941	0.9832
1.00	124I	2.0e+05	4096	2.0e+05	1.0022	1.0025	0.1020	10.1726	1.0089
1.00	145Eu	3.0e+05	4096	3.0e+05	0.9893	0.9988	0.0748	7.4923	0.9987
1.00	165Tm	1.0e+05	4096	1.0e+05	1.0019	0.9919	0.1030	10.3797	0.9869
1.00	71As	2.5e+04	4096	2.5e+04	0.9821	0.9640	0.3554	36.8659	0.9566
1.00	44Sc	2.0e+05	4096	2.0e+05	1.0005	1.0030	0.1068	10.6451	1.0013
1.00	57Ni	3.0e+05	4096	3.0e+05	0.9991	0.9956	0.0566	5.6819	0.9956
T #9									
1.00	103Ru	1.0e+05	4096	1.0e+05	0.9867	0.9858	0.1292	13.1090	0.9942
2.00	99Mo	5.0e+04	4096	1.0e+05	2.0069	1.9932	0.1603	8.0430	1.9848
0.50	124I	2.0e+05	4096	1.0e+05	0.4995	0.5002	0.0714	14.2771	0.5024
0.33	145Eu	3.0e+05	4096	1.0e+05	0.3271	0.3311	0.0458	13.8405	0.3311
1.00	165Tm	1.0e+05	4096	1.0e+05	1.0079	1.0014	0.0949	9.4736	0.9999
4.00	71As	2.5e+04	4096	1.0e+05	3.9750	3.9249	0.5470	13.9365	3.8822
0.50	44Sc	2.0e+05	4096	1.0e+05	0.4980	0.5004	0.0728	14.5486	0.5005
0.33	57Ni	3.0e+05	4096	1.0e+05	0.3351	0.3349	0.0374	11.1725	0.3349
T #10									
1.00	103Ru	1.0e+05	4096	1.0e+05	0.9867	0.9858	0.0000	0.0000	0.9942
2.00	99Mo	5.0e+04	4096	1.0e+05	2.0069	1.9928	0.0000	0.0000	1.9837
0.50	124I	2.0e+05	4096	1.0e+05	0.4995	0.5002	0.0000	0.0000	0.5024
0.33	145Eu	3.0e+05	4096	1.0e+05	0.3271	0.3311	0.0000	0.0000	0.3311
1.00	165Tm	1.0e+05	4096	1.0e+05	1.0079	1.0012	0.0000	0.0000	0.9997
4.00	71As	2.5e+04	4096	1.0e+05	3.9750	3.9242	0.0000	0.0000	3.8819

<u>IDEAL</u> a's	SRCE	<u>SRCE</u> AMT	CNTS	<u>DATA</u> AMT	A's	SIG A's	SIGMA1	%ERROR	STRPD a's
0.50	44Sc	2.0e+05	4096	1.0e+05	0.4980	0.5005	0.0000	0.0000	0.5005
0.33	57Ni	3.0e+05	4096	1.0e+05	0.3351	0.3349	0.0000	0.0000	0.3349
0.00	118Te	4.0e+05	4096		2.3675	3.8982	64.4112	1652.3313	-47.1164
T #11									
1.00	103Ru	1.0e+05	4096	1.0e+05	0.9868	0.9856	0.1292	13.1117	0.9940
2.00	99Mo	5.0e+04	4096	1.0e+05	2.0069	1.9923	0.1606	8.0622	1.9836
0.50	124I	2.0e+05	4096	1.0e+05	0.4996	0.4998	0.0714	14.2886	0.5019
0.33	145Eu	3.0e+05	4096	1.0e+05	0.3272	0.3308	0.0458	13.8530	0.3306
1.00	165Tm	1.0e+05	4096	1.0e+05	1.0081	1.0007	0.0949	9.4802	0.9992
4.00	71As	2.5e+04	4096	1.0e+05	3.9752	3.9236	0.5470	13.9411	3.8830
0.50	44Sc	2.0e+05	4096	1.0e+05	0.4981	0.5002	0.0728	14.5544	0.5005
0.33	57Ni	3.0e+05	4096	1.0e+05	0.3351	0.3347	0.0374	11.1791	0.3347
0.00	188Ir	4.0e+05	4096		-0.0004	0.0008	0.0000	0.0000	0.0008
T #12									
1.00	103Ru	1.0e+05	4096	1.0e+05	0.9868	0.9855	0.1292	13.1130	0.9939
2.00	99Mo	5.0e+04	4096	1.0e+05	2.0070	1.9922	0.1606	8.0626	1.9835
0.50	124I	2.0e+05	4096	1.0e+05	0.4996	0.4998	0.0721	14.4280	0.5019
0.33	145Eu	3.0e+05	4096	1.0e+05	0.3273	0.3308	0.0458	13.8530	0.3306
1.00	165Tm	1.0e+05	4096	1.0e+05	1.0083	1.0006	0.0954	9.5337	0.9990
4.00	71As	2.5e+04	4096	1.0e+05	3.9757	3.9231	0.5477	13.9615	3.8823
0.50	44Sc	2.0e+05	4096	1.0e+05	0.4981	0.5002	0.0728	14.5544	0.5005
0.33	57Ni	3.0e+05	4096	1.0e+05	0.3351	0.3347	0.0374	11.1791	0.3347
0.00	188Ir	4.0e+05	4096		-0.0003	0.0007	0.0000	0.0000	0.0007
0.00	153Tb	1.0e+05	4096		-0.0027	0.0008	0.0387	4841.2292	0.0008
T #13									
1.00	103Ru	1.0e+05	4096	1.0e+05	0.9862	0.9829	0.1311	13.3430	0.9918
2.00	99Mo	5.0e+04	4096	1.0e+05	2.0117	1.9967	0.1664	8.3354	1.9733
0.50	124I	2.0e+05	4096	1.0e+05	0.4995	0.4992	0.0735	14.7205	0.5021
0.33	145Eu	3.0e+05	4096	1.0e+05	0.3297	0.3328	0.0490	14.7205	0.3343
1.00	165Tm	1.0e+05	4096	1.0e+05	1.0097	1.0006	0.1015	10.1428	0.9945
4.00	71As	2.5e+04	4096	1.0e+05	3.9842	3.9560	0.5696	14.3974	3.9352
0.50	44Sc	2.0e+05	4096	1.0e+05	0.4955	0.4967	0.0755	15.2000	0.4963
0.33	57Ni	3.0e+05	4096	1.0e+05	0.3348	0.3330	0.0387	11.6306	0.3335
0.25	188Ir	4.0e+05	4096	1.0e+05	0.2476	0.2477	0.0224	9.0273	0.2477
1.00	153Tb	1.0e+05	4096	1.0e+05	0.9935	0.9922	0.1212	12.2197	0.9960
T #14									
0.10	241Am	1.0e+06	4096	1.0e+05	-0.0006	-0.0003	0.0000	0.0000	0.0000
4.00	71As	2.5e+04	4096	1.0e+05	3.9695	3.9257	0.5477	13.9522	3.8259

IDEAL a's	SRCE	SRCE AMT	CNTS	DATA AMT	A's	SIG A's	SIGMA1	%ERROR	STRPD a's
0.00	60Co	1.0e+06	4096		0.0000	0.0000	0.0000		0.0000
0.00	132Cs	1.0e+06	4096		0.0000	0.0000	0.0000		0.0001
0.00	137Cs	1.0e+06	4096		-0.0001	-0.0010	0.0000	0.0000	0.0000
0.33	145Eu	3.0e+05	4096	1.0e+05	0.3275	0.3284	0.0000	0.0000	0.3302
0.50	124I	2.0e+05	4096	1.0e+05	0.4977	0.4971	0.0000	0.0000	0.5008
0.00	188Ir	4.0e+05	4096		-0.0003	0.0006	0.0000	0.0000	0.0006
2.00	99Mo	5.0e+04	4096	1.0e+05	2.0072	1.9930	0.0000	0.0000	1.9860
0.33	57Ni	3.0e+05	4096	1.0e+05	0.3319	0.3319	0.0000	0.0000	0.3339
1.00	103Ru	1.0e+05	4096	1.0e+05	0.9871	0.9860	0.0000	0.0000	0.9935
0.50	44Sc	2.0e+05	4096	1.0e+05	0.4985	0.4955	0.0000	0.0000	0.4997
0.00	153Tb	1.0e+05	4096		-0.0021	0.0008	0.0000	0.0000	0.0008
0.00	118Te	4.0e+05	4096		2.7126	3.9228	64.4368	1642.6224	-46.8088
1.00	165Tm	1.0e+05	4096	1.0e+05	1.0089	1.0010	0.0000	0.0000	0.9973
0.00	22Na	1.0e+04	4096		0.0543	0.0170	0.3162	1860.1633	0.0782
0.00	26Al	2.0e+04	4096		0.0181	0.0033	0.0000	0.0000	0.0101
0.00	47Ca	3.0e+04	4096		-0.0081	-0.0005	0.0000	0.0000	0.0000
0.00	48Sc	6.0e+04	4096		-0.0005	-0.0029	0.0000	0.0000	0.0005
0.00	48V	9.0e+04	4096		-0.0002	0.0021	0.0000	0.0000	0.0000
0.00	52Mn	2.0e+06	4096		-0.0001	0.0000	0.0000		0.0000
0.00	59Fe	4.0e+06	4096		0.0000	0.0000	0.0000		0.0000
0.00	56Co	5.0e+04	4096		0.0002	0.0304	0.0000	0.0000	0.0016
0.00	57Co	7.0e+04	4096		-0.0027	-0.0021	0.0000	0.0000	0.0000
0.00	65Zn	8.0e+04	4096		-0.0029	-0.0008	0.0000	0.0000	0.0000
T #15									
0.00	118Ag	5.0e+04	4096		0.0149	0.0848	0.0000	0.0000	0.1372
1.00	26Al	1.0e+04	4096	1.0e+04	-0.0416	0.4749	0.0000	0.0000	0.5632
0.02	241Am	1.0e+06	4096	2.0e+04	0.0112	0.0180	0.0000	0.0000	0.0182
1.00	71As	3.0e+04	4096	3.0e+04	1.0290	1.0683	0.0000	0.0000	1.0332
1.60	82Br	5.0e+05	4096	8.0e+05	1.5937	1.6069	0.0000	0.0000	1.6075
1.33	47Ca	3.0e+04	4096	4.0e+04	1.6367	1.6985	0.0000	0.0000	1.7484
1.00	56Co	5.0e+04	4096	5.0e+04	1.0391	1.1058	0.0000	0.0000	1.2534
0.00	57Co	7.0e+04	4096		0.0142	0.0247	0.0000	0.0000	0.0251
0.00	60Co	1.0e+06	4096		0.0018	0.0050	0.0000	0.0000	0.0051
0.06	132Cs	1.0e+06	4096	6.0e+04	0.0590	0.0606	0.0000	0.0000	0.0607
0.00	137Cs	1.0e+06	4096		0.0007	0.0015	0.0000	0.0000	0.0015
3.00	68Cu	2.0e+05	4096	6.0e+05	3.0603	3.1186	0.0000	0.0000	3.1451
0.23	145Eu	3.0e+05	4096	7.0e+04	0.2166	0.2256	0.0000	0.0000	0.2258
0.02	59Fe	4.0e+06	4096	8.0e+04	0.0176	0.0185	0.0000	0.0000	0.0185
2.33	64Ga	3.0e+05	4096	7.0e+05	3.4727	3.5528	0.0000	0.0000	3.5616
0.00	65Ge	4.0e+05	4096		-0.0342	-0.0115	0.0000	0.0000	0.0000
0.00	124I	2.0e+05	4096		-0.0185	-0.0063	0.0000	0.0000	0.0000

IDEAL a's	SRCE	SRCE AMT	CNTS	DATA AMT	A's	SIG A's	SIGMA1	%ERROR	STRPD a's
0.00	188Ir	4.0e+05	4096		0.0085	0.0154	0.0000	0.0000	0.0154
1.50	79Kr	6.0e+05	4096	9.0e+05	1.4611	1.5036	0.0000	0.0000	1.5072
0.04	52Mn	2.0e+06	4096	9.0e+04	0.0451	0.0459	0.0000	0.0000	0.0459
0.00	91Mo	1.0e+04	4096		-1.4016	-0.2021	0.0000	0.0000	0.0000
0.00	99Mo	5.0e+04	4096		-0.0075	0.0148	0.0000	0.0000	0.0154
10.00	22Na	1.0e+04	4096	1.0e+05	10.3988	10.3389	3.1623	30.5862	9.5850
0.67	57Ni	3.0e+05	4096	2.0e+05	0.7179	0.7073	0.0000	0.0000	0.0709
50.00	99Pd	4.0e+04	4096	2.0e+06	49.9271	49.7709	0.0000	0.0000	49.6117
0.00	88Rb	7.0e+05	4096		0.0909	1.6399	0.0000	0.0000	1.7787
0.00	96Rh	3.0e+04	4096		-0.1318	-0.0729	0.0000	0.0000	0.0000
0.00	103Ru	1.0e+05	4096		0.0128	0.0291	0.0000	0.0000	0.0298
0.00	44Sc	2.0e+05	4096		-0.0538	-0.0501	0.0000	0.0000	0.0000
0.12	48Sc	4.0e+06	4096	5.0e+05	8.4066	8.4391	0.0000	0.0000	8.4872
0.00	83Sr	8.0e+05	4096		0.0146	0.0293	0.0000	0.0000	0.0287
0.00	153Tb	1.0e+05	4096		0.0235	0.0752	0.0000	0.0000	0.0838
150.00	92Tc	2.0e+04	4096	3.0e+06	150.1731	147.7467	0.0000	0.0000	147.4929
0.00	118Te	4.0e+05	4096		167.3739	196.6716	344.7898	175.3124	-1090.7000
0.00	53Ti	1.0e+05	4096		-0.0074	0.0154	0.0000	0.0000	0.0167
66.67	208Tl	6.0e+04	4096	4.0e+06	66.4954	65.4287	0.0000	0.0000	65.3497
0.00	165Tm	1.0e+05	4096		-0.2546	-0.1407	0.0000	0.0000	0.0000
3.33	48V	9.0e+04	4096	3.0e+05	3.3149	3.3415	0.0000	0.0000	3.3118
1.11	88Y	9.0e+05	4096	1.0e+06	1.0951	0.8818	0.0000	0.0000	0.8552
5.00	65Zn	8.0e+04	4096	4.0e+05	4.9504	4.9718	0.0000	0.0000	5.0710
0.00	87Zr	1.0e+06	4096		0.0253	0.0052	0.0000	0.0000	0.0000
T #16									
1.00	26Al	1.0e+04	4096	1.0e+04	0.2501	0.5411	0.7210	133.2418	0.4194
0.02	241Am	1.0e+06	4096	2.0e+04	0.0105	0.0189	0.0755	399.4621	0.0192
1.00	71As	3.0e+04	4096	3.0e+04	1.0259	1.038	1.1670	112.4282	1.0635
1.60	82Br	5.0e+05	4096	8.0e+05	1.5901	1.6038	0.0529	3.2994	1.6042
1.33	47Ca	3.0e+04	4096	4.0e+04	1.5112	1.7323	1.0093	58.2611	2.4563
1.00	56Co	5.0e+04	4096	5.0e+04	1.0171	1.1289	0.3124	27.6738	1.5913
0.06	132Cs	1.0e+06	4096	6.0e+04	0.0579	0.0599	0.0200	33.3890	0.0599
3.00	68Cu	2.0e+05	4096	6.0e+05	3.0453	3.1312	0.2891	9.2341	3.1286
0.23	145Eu	3.0e+05	4096	7.0e+04	0.2046	0.2269	0.0894	39.4194	0.2283
0.02	59Fe	4.0e+06	4096	8.0e+04	0.0167	0.0187	0.0100	53.4759	0.0187
2.33	64Ga	3.0e+05	4096	7.0e+05	3.4735	3.5884	0.2049	5.7112	3.5936
1.50	79Kr	6.0e+05	4096	9.0e+05	1.4539	1.4986	0.1349	9.0022	1.5012
0.04	52Mn	2.0e+06	4096	9.0e+04	0.0445	0.0458	0.0000	0.0000	0.0458
10.00	22Na	1.0e+04	4096	1.0e+05	10.6424	10.2046	2.8842	28.2639	8.7943
0.67	57Ni	3.0e+05	4096	2.0e+05	0.7097	0.7163	0.1034	14.4410	0.7221
50.00	99Pd	4.0e+04	4096	2.0e+06	49.9201	49.7508	0.9481	1.9056	49.6413

IDEAL a's	SRCE	SRCE AMT	CNTS	DATA AMT	A's	SIG A's	SIGMA1	%ERROR	STRPD a's
0.12	48Sc	4.0e+06	4096	5.0e+05	8.3643	8.4513	0.6683	7.9074	8.3567
150.00	92Tc	2.0e+04	4096	3.0e+06	150.0957	147.8149	1.6221	1.0974	147.3022
66.67	208Tl	6.0e+04	4096	4.0e+06	66.4769	65.7032	1.0081	1.5343	65.7740
3.33	48V	9.0e+04	4096	3.0e+05	3.3349	3.3041	0.5968	18.0632	3.3113
1.11	88Y	9.0e+05	4096	1.0e+06	1.1052	1.1217	0.0458	4.0854	1.1218
5.00	65Zn	8.0e+04	4096	4.0e+05	4.9227	4.9615	0.7902	15.9264	4.8596
T #17									
1.00	149Pm	1.0e+05	4096	1.0e+05	0.9533	1.0233	0.6650	64.9840	1.1812
10.00	22Na	1.0e+04	4096	1.0e+05	9.9722	9.9727	0.4805	4.8184	9.9727
0.25	118Te	4.0e+05	4096	1.0e+05	0.7770	0.7730	27.6702	3579.5882	0.0573
T #18									
0.25	118Te	4.0e+05	4096	1.0e+05	0.2312	0.2316	5.6721	2449.1040	0.1880
1.00	149Pm	1.0e+05	4096	1.0e+05	1.0000	0.9999	0.3484	34.8460	0.9999
T #19									
1.00	232Th	2.0e+05	512	2.0e+05	0.9986	0.9971	0.7338	73.5892	0.9923
1.00	40K	2.0e+05	512	2.0e+05	0.9982	0.9989	0.3002	30.0497	0.9989
1.00	238U	2.0e+05	512	2.0e+05	0.9993	0.9990	0.2762	27.6501	0.9990